Exercise 4.6. Carry out the ket recipe with , final observable , initial state , and observable measured at time *t*. Find the possible outcomes and their probabilities of occurrence.

Solution. I believe the initial state and final observables were inadvertently reversed in the problem statement because reversing them makes this exercise the natural culmination of a problem started in Exercise 3.4 (represent  in spherical coordinates), and continued in 4.5 (for , find the energy eigenvectors and eigenvalues ). It also becomes a means to confirm the results of Exercise 4.4 that the 3-vector operator  precesses clockwise around the direction of the magnetic field. The problem as stated, as others have shown, results in  unchanging over time, a not very interesting result. I provide both results, doing the reversed problem 1st.

I thus assume an initial state  (corresponds to observable ) and a final observable  (corresponds to state ). Moreover, the following spin formulas become valid because they were developed under the assumption that system is prepared in some state  and then measured in the up direction:



Step 1. Find *H*. 

Step 2. Prepare a state vector. The state vector that corresponds to the observable  is

 .

Step 3. Find the energy eigenvalues and eigenvectors of *H*. From Exercise 4.5 we learned that for a generic direction  that  has the following energy eigenvalues and corresponding eigenvectors:



, and for  we get that . Hence

 ✔

.

 is a unit vector for any value of . However, in Exercise 4.5 it was shown that  and  where  are the eigenvectors of . Since  and  we have that

 (and consequently . ✔

Step 4. Calculate**.**





Step 5. .



Sanity check: 

 in Step 2 is same as in Step 5 ✔

Step 6. Expand  in terms of 



Step 7. Replace  in (6) with .



Step 8. Specify a new observable at time *t*, compute its eigenvalues  and eigenvectors , and calculate the probabilities of the outcomes.

According to the problem statement, the observable  is measured at time *t*. Thus . From Exercise 3.4,

, ,

 , and .

Thus





Check:  ✔

Another check: Both probabilities vary between 0 and 1. ✔

The conclusion is that  varies as a sinusoidal wave over time between 0 and 1 with a mean value of .

Similarly,  varies as a sine wave between 0 and 1, and  .

For the record, working the problem as stated yields:

1. 
2. 
3. 
4. 



1. 
2. 
3. 
4. Compute the eigenvectors and eigenvalues of  at time t:

.

From Exercise 3.4,



and

.

At time *t:*





and





** is unchanging over time (i.e., P(1) and P(2) are not expressed in terms of time).