Exercise 4.6. Carry out the ket recipe with , final observable , initial state , and observable measured at time *t*. Find the possible outcomes and their probabilities of occurrence.

Solution. This exercise is the culmination of a problem started in Exercise 3.4 and continued in 4.4 and 4.5.

There is an unfortunate choice of wording in the 4.6 problem statement. We should not confuse "initial state" in the problem statement with "initial state" in step 2 of the ket recipe. They are quite different.

The term "initial state" in step 2 is simply the state vector that corresponds to the selected observable. However, the phrase "initial state " in the wording of this problem simply means that as developed in chapter 2, formula (2.13), we should use the following additional formulas:



Step 1. Find *H*. 

Step 2. Prepare a state vector. The state vector that corresponds to the observable  is

 .

Step 3. Find the energy eigenvalues and eigenvectors of *H*. From Exercise 4.5 we learned that for a generic direction  that  has the following energy eigenvalues and corresponding eigenvectors:



, and for  we get that . Hence

 ✔

.

 is a unit vector for any value of **. However, in Exercise 4.5 it was shown that  and  where  are the eigenvectors of . Since  and  we have that

 (and consequently ** = **.

Step 4. Calculate**.**





Step 5. .



Sanity check: 

 in Step 2 is same as in Step 5 ✔

Step 6. Expand  in terms of 



Step 7. Replace  in (6) with .



Step 8. Specify a new observable at time *t*, compute its eigenvalues  and eigenvectors .

According to the problem statement, the observable  is measured at time *t*.

Thus . From Exercise 3.4, , ,  , and .

Thus





Check:  ✔

Another check: Both probabilities vary between 0 and 1. ✔

The conclusion is that  varies as a sinusoidal wave over time with a mean value of .

We could also have shown a similar result for . Of course  for all *t*.